

# Considerations of Cosmic Acceleration

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## Abstract

I discuss a solution to the dark energy problem, which arises when the visible universe is approximated by a black hole, in a quasi-static asymptotically-flat approximation. Using data, provided by WMAP7, I calculate the Schwarzschild radius  $r_S$  and compare to the measured physical radius of the visible universe, bounded by the surface of last scatter. The ratio,  $\epsilon(t_0) = r/r_S$  is found to be comparable to  $\epsilon = 1$ , as allowed by the holographic principle. The measurement of a shift parameter,  $\sigma$ , introduced by Bond, Efstathiou and Tegmark in 1997, plays an important role in the accuracy of the calculation. The approximation leads to a surprisingly small discrepancy, presumably explicable by the de Sitter, and expanding, nature of the actual universe.

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# 1 Introduction

In fundamental theoretical physics, there was, at the beginning of the twenty-first century, an impossible seeming problem. The problem is the dark energy in cosmology, comprising some seventy percent of the universe.

Another cosmological problem, closely entwined with the dark energy problem, is the question well posed, now almost eight decades ago, by Tolman[1] as to whether one can construct a consistent cyclic model, given the seemingly contradictory constraint imposed by the second law of thermodynamics. The most developed solution of this Tolman conundrum is that suggested in [2].

The most important observational advance in cosmology since the early studies of cosmic expansion in the 1920s was the dramatic and, at that time, surprising discovery in the waning years of the twentieth century that the expansion is accelerating. This was first announced in February 1998, and it was based on the concordance of two groups' data on Supernovae Type 1A [3, 4]

Many subsequent experiments concerning the Cosmic Microwave Background (CMB), Large Scale Structure (LSS) and other measurements have all confirmed the 1998 claim. I therefore adopt the position that the accelerated expansion rate is an observed fact.

Assuming general relativity, together with the cosmological principle of homogeneity and isotropy, the scale factor  $a(t)$  in the FRW metric satisfies [5, 6] the Friedmann-Lemaître equation, with different energy density components subsumed into  $\rho$ :

$$H(t)^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right) \rho. \quad (1)$$

I normalize  $a(t_0) = 1$  at the present, time  $t = t_0$ , and  $\rho$  is an energy density source which drives the expansion of the universe. Two established contributions to  $\rho$  are  $\rho_m$  from matter (including dark matter) and  $\rho_\gamma$  radiation, so that

$$\rho \supseteq \rho_m + \rho_\gamma \quad (2)$$

with  $\rho_m(t) = \rho_m(t_0)a(t)^{-3}$  and  $\rho_\gamma(t) = \rho_\gamma(t_0)a(t)^{-4}$ .

For the observed accelerated expansion, a phenomenological approach is to add to the sources, in Eq.(1), a dark energy term  $\rho_{DE}(t)$  with

$$\rho_{DE}(t) = \rho_{DE}(t_0)a(t)^{-3(1+\omega)}, \quad (3)$$

where  $\omega = p/\rho$  is the equation of state. For the case  $\omega = -1$ , as for a cosmological constant  $\Lambda$ , and if one discards the matter and radiation terms which are smaller than the dark energy term, one can easily integrate the Friedmann-Lemaître equation to find

$$a(t) = a(t_0) e^{Ht} \quad (4)$$

where  $\sqrt{3}H = \sqrt{\Lambda} = \sqrt{8\pi G\rho_{DE}}$ .

By differentiation of Eq. (4) with respect to time  $p$  times, one obtains for the  $p^{th}$  derivative

$$\frac{d^p}{dt^p}a(t)|_{t=0} = (H)^p \quad (5)$$

Therefore, if  $\Lambda$  is positive, as in a De Sitter geometry, not only is the acceleration ( $p = 2$ ) positive and non-zero, but so are the jerk ( $p = 3$ ), the snap ( $p = 4$ ), the crackle ( $p = 5$ ), the pop ( $p = 6$ ) and all derivatives for  $p \geq 7$ .

The insertion of the dark energy term Eq. (3) in Eq. (1) works very well as a part of the  $\Lambda$ CDM model. However, it is an *ad hoc* procedure which gives no insight into what dark energy is.

## 2 Dark Energy Problem

With this background, I shall now move to a different explanation for the accelerated expansion which obviates any dark energy, including any need for a cosmological constant.

The following considerations may initially appear to be trivial, circular, tautological or some combination thereof. It appears, nevertheless, by hindsight that the 1998 discovery of cosmic acceleration could have been far less surprising theoretically had one previously thought of the  $\Lambda = 0$  universe as a black hole.

I now adopt this different approach, with no dark energy, where instead the central role is played by the assumption of the holographic principle[7, 8] ( see also [9]) and by the overriding concept of entropy.

The essential assumption is the aforementioned holographic principle, by which I understand that all the information about the universe is encoded on its two-dimensional surface. What this implies is, however unlikely it seems and however contrary to everyday experience that the three-dimensional world I apparently observe is somehow an illusion. This can lead to a reinterpretation of the cosmic acceleration, and possibly the most dramatic new insight into gravity in over three centuries.

Consider the Schwarzschild radius ( $r_s$ ), and the physical radius ( $R$ ), of the Sun ( $\odot$ ). They are  $(r_s)_{\odot} = 3km$  and  $R_{\odot} = 800,000km$ . Their ratio is  $(\rho)_{\odot} \equiv (R/r_s)_{\odot} = 2.7 \times 10^5$ . One can readily check that for the Earth or for the Milky Way the ratio  $\rho = (R/r_s)$  is likewise much larger than one:  $\rho \gg 1$ . Such objects are nowhere close to being black hole. Now consider the visible universe (VU) up to the surface of last scatter. As we shall discuss later, with mass  $M_{VU} \sim 5.5 \times 10^{23} M_{\odot}$ , it has  $(r_s)_{VU}$  surprisingly close to  $(r)_{VU} \sim 14Gpc$ , even with the approximations of a quasi-static and asymptotically-flat universe. The visible universe, within which we all live, is well approximated by a black hole. The following addressing of the dark energy problem follows from this observation.

At the horizon, there is a PBH temperature [10, 11, 12],  $T_{\beta}$ , which I can estimate as

$$T_{\beta} = \frac{\hbar}{k_B} \frac{H}{2\pi} \sim 3 \times 10^{-30} K. \quad (6)$$

This temperature of the horizon information screen leads to a concomitant FDU acceleration [13, 14, 15]  $a_{Horizon}$ , outward, of the horizon given by the relationship

$$a_{Horizon} = \left( \frac{2\pi c k_B T_\beta}{\hbar} \right) = cH \sim 10^{-9} m/s^2. \quad (7)$$

When  $T_\beta$  is used in Eq. (7), I arrive at a cosmic acceleration which is essentially in agreement with the observations[3, 4].

From this viewpoint, the dark energy is non-existent. Instead there is a consequence of the second law of thermodynamics, acting to create the appearance of a dark energy component of the driving density on the right-hand-side of the Friedman-Lemaître equation, Eq.(1).

I have discussed a theory underlying the accelerated expansion of the universe based on entropy. This approach provides a physical understanding of the acceleration phenomenon which was lacking in the description as dark energy.

The entropy of the universe has received some recent attention [16], in part because it relates to the feasibility of constructing a consistent cyclic model. For example, the cyclic model in [2], assuming its internal consistency will indeed be fully confirmed, provides the solution to a difficult entropy question originally posed, seventy-five years earlier, by Tolman [1]. The accelerated expansion rate is no longer surprising. It is the inevitable consequence of information storage on the surface of the visible universe.

This solution of the dark energy problem not only solves a cosmological problem but also casts a completely new light on the nature of the gravitational force [17, 18, 19, 20]. Since the expansion of the universe, including the acceleration thereof, can only be a gravitational phenomenon, I arrive at the viewpoint that gravity is a classical result of the second law of thermodynamics. This means that gravity cannot be regarded as on the same footing with the electroweak and strong interactions. Only if one assumes higher spatial dimensions, it can be argued[21] that non-gravitational interactions are also be emergent.

Although this can be the most radical change in gravity theory for three centuries, it is worth emphasizing that general relativity and its classical tests remain unscathed, as does the prediction of gravitational waves.

The result calls into question almost all of the work done on quantum gravity, since the discovery of quantum mechanics. For gravity, there is no longer necessity for a graviton. In the case of string theory, the principal motivation[22, 23] for the profound and historical suggestion by Scherk and Schwarz that string theory be reinterpreted, not as a theory of the strong interaction, but instead as a theory of the gravitational interaction, came from the natural appearance of a massless graviton in the closed string sector.

This is not saying that string theory is dead. What it is saying is, that string theory cannot be a theory of the fundamental gravitational interaction, since there is no fundamental gravitational interaction.

The way this new insight emerged, as a solution of the dark energy problem itself, was as a natural line of thought, following the discovery of a cyclic model in [2], and the subsequent investigations [24, 25, 26, 27, 28] of the entropy of the universe, including a possible candidate for dark matter[26, 29]. It is also strongly overlapping with the discussions in [17, 18].

Another ramification, of such a solution of the dark energy, problem is the status, fundamental versus emergent, of the three spatial dimensions, that we all observe every day. Because the solution assumes the holographic principle[7, 8], at least one spatial dimension appears as emergent. Regarding the visible universe as a sphere, with radius of about 14 Gpc, the emergent space dimension is then, in spherical polar coordinates, the radial coordinate, while the other two coordinates, the polar and azimuthal angles, remain fundamental. Physical intuition, related to the isotropy of space, may suggest that, if one space dimension is emergent, then so must be all three. This merits further investigation, and may require a generalization of the holographic principle in [7, 8]. On the other hand, a fundamental time coordinate is useful in dynamics.

Of course, this present discussion of cosmic acceleration, is merely one small step towards the ultimate goal, of a cyclic model, in which time never begins or ends.

### 3 Holographic Principle

An interesting and profound idea about the degrees of freedom describing gravity, is the holographic principle [7, 8].

For the case of a sphere, with mass  $M$ , of radius  $r$ , where  $r$  will be the co-moving radius for the expanding universe, a form, of the holographic principle, states that the ration  $\epsilon = r_S/r$ , has an upper limit equal to that of a black hole, *i.e.*

$$\epsilon = \left( \frac{r_S}{r} \right) \leq 1. \quad (8)$$

With  $G$  as Newton's constant,  $r_S = 2GM$  is the Schwarzschild radius. It is interesting, from the viewpoint of the physical understanding of the visible universe, to use accurate observational data to check, whether the simplified, and non-covariant, Eq.(8) is satisfied at the present time,  $t = t_0$ , and in the past, cognizant that, with dark energy, if  $r$  sufficiently increases, Eq.(8) will eventually be violated, if the universe expands exponentially quickly, and if one assumes the approximations used here. Note that, at the time of [7], before dark energy, if Eq.(8) is now satisfied, one might expect it to remain so.

The holographic principle is supported, by string theory. The AdS/CFT correspondence [30] is an explicit realization of Eq.(8), and so, apart from the non-trivial subtlety that our universe is dS, not AdS, from the viewpoint of string theory, there is every reason to believe the holographic principle, and to wish to check Eq.(8). It is related to recent considerations of the entropy of the universe [17, 27, 28].

However, physics is an empirical science, and therefore the scientific method dictates that we should find a physical example, in which Eq.(8) can be calculated. The result, reported here, is that a detailed and accurate check of Eq.(8), as applied to the visible universe, fails, by a statistically-significant amount, although in the past, a few billion years ago, it was satisfied.

I should define, precisely, what is meant by the visible universe. It is the sphere centered for convenience at the Earth with radius  $d_A(Z^*) = 14.0 \pm 0.1 Gpc$ . The value of  $d_A(Z^*)$  is

the particle horizon corresponding to the recombination red shift  $Z^* = 1090 \pm 1$ , and is measured directly by WMAP7 [31], without needing the details of the expansion history. Thus, “visible” means with respect to electromagnetic radiation.

## 4 The Visible Universe

The motion is that the visible universe, so defined, is a physical object which should be subject to the holographic principle. It is an expanding, rather than a static, object, yet my understanding is that the principle, at least in its covariant form, is still expected to be valid.

I shall use the notation employed by the WMAP7 paper [31], from which all observational data are taken.

The present age,  $t_0$ , of the universe is measured to be

$$t_0 = 13.75 \pm 0.13 Gy \tag{9}$$

The comoving radius,  $d_A(Z^*)$ , of the visible universe, is, likewise, measured to one percent accuracy:

$$d_A(Z^*) \equiv (1 + Z^*)D_A(Z^*) = c \int_{t^*}^{t_0} \frac{dt}{a(t)} = 14.0 \pm 0.1 Gpc, \tag{10}$$

where it is noted that the measurement, of  $d_A(Z^*)$ , does not require knowledge, of the expansion history,  $a(t)$ , for  $t^* \leq t \leq t_0$ .

The critical density,  $\rho_c$ , is provided by the formula

$$\rho_c = \left( \frac{3H_0^2}{8\pi G} \right), \tag{11}$$

whose value depends on  $H_0$ , as does the total, baryonic plus dark, matter density,  $\rho_m$



$$\rho_m \equiv \Omega_m \rho_c. \quad (12)$$

Because the error on the Hubble parameter,  $H_0$ , is several per cent, it is best to avoid  $H_0$ , in checking the holographic principle.

The mass of the matter,  $M(Z^*)$ , contained in the visible universe, is first estimated, to be augmented later, as

$$M(Z^*) = \frac{4\pi}{3} d_A(Z^*)^3 \rho_m \quad (13)$$

which gives  $M(Z^*) \sim 5.5 \times 10^{23} M_\odot$ , consistent with the oft-used estimates of  $\sim 10^{11}$  galaxies, each of mass  $\sim 10^{12} M_\odot$  including dark matter. The Schwarzschild radius,  $r_S(Z^*)$ , is given by  $R_S(Z^*) \equiv 2GM(Z^*)$ .

Collecting results enables the desired accurate check of the simplified holographic principle, which compares the radius,  $r$ , for the visible universe with that for a black hole,  $r_S$  of the same mass.

A shift parameter,  $\sigma$ , was defined by Bond, Efstathiou and Tegmark (BET) in [32], as

$$\sigma = \frac{\sqrt{\Omega_m H_0^2}}{c} (1 + Z^*) D_A(Z^*), \quad (14)$$

which was, with great prescience, introduced by BET, as a dimensionless quantity, to be measured, accurately, by CMB observations.

This BET shift parameter,  $\sigma$ , of Eq. (14), is given in [31], as (Note that  $\sigma$  is designated  $R$  in [31])

$$\sigma = 1.725 \pm 0.018. \quad (15)$$

A little algebra shows that the BET shift parameter  $\sigma$  provides the most accurate available check, of the holographic principle, by virtue of the result

$$\epsilon(t_0) = \left[ \frac{r_S}{r} \right] \equiv \sigma^2 = 2.976 \pm 0.062 \quad (16)$$

which is surprisingly close to the saturation of Eq.(8). The derivation of Eq.(16) is as follows:

$$\epsilon = \left( \frac{r_S}{r} \right) \quad (17)$$

$$= \left( \frac{1}{(1 + Z^*)D_A(Z^*)} \right) \left( 2G \cdot \frac{4\pi}{3} (1 + Z^*)^3 D_A(Z^*)^3 \Omega_m \frac{3H_0^2}{8\pi G} \right) \quad (18)$$

$$= \Omega_m H_0^2 (1 + Z^*)^2 D_A(Z^*)^2 \quad (19)$$

$$= \sigma^2 = 2.976 \pm 0.062 \quad Q.E.D. \quad (20)$$

I used the definition in Eq.(14) for this calculation. It is very fortunate that the square of  $\sigma$ , defined in Eq. (14), agrees with the calculation of  $\epsilon$ , in Eq. (16).

This is suggestive that the expression, Eq.(13), overestimates somewhat the mass because the density is smaller at the sphere's center, where the observer is. More accurately, one can be to take into account this variation of density with redshift by using instead

$$\begin{aligned} M(Z^*) &= \int_0^{d_A(Z^*)} dr 4\pi r^2 \rho_m \left( \frac{r}{d_A(Z^*)} \right)^3 \\ &= \frac{1}{2} \frac{4\pi d_A(Z^*)^3 \rho_m}{3}, \end{aligned} \quad (21)$$

which gives  $M(Z^*)m \sim 2.7 \times 10^{23} M_\odot$  and reduces the estimate of  $\epsilon(t_0)$  by a factor two to

$$\epsilon(t_0) = 1.48 \pm 0.03, \quad (22)$$

with a discrepancy of only 48 percent which is surprisingly small compared to the size of either of the radii, both of which are  $\sim 10^{61}$  in natural (Planck) units. The mass of the visible universe in Eq.(21) for the volume in asymptotically-flat space is a phenomenological estimate, but the best available.

## 5 Discussion

To my knowledge, the visible universe is, at present, the only physical object, for which it is possible to calculate, and compare with experiment, or observation, the simplified holographic principle.

The result Eq. (22) is surprisingly close to  $\epsilon = 1$ . The departure from Eq.(8) can presumably be ascribed to the two assumptions of (i) asymptotic flatness and (ii) a quasi-static universe. It is therefore a very interesting challenge to relax these assumptions and further to use the precision WMAP7 data to attempt to confirm that the holographic principle, Eq. (8), is precisely saturated by the observable universe.

Such a calculational check is under way. For example, one may try replacing the Schwarzschild metric by the McVittie metric[33] which accommodates both a cosmological constant and an expanding universe. Whether this can confirm even more accurately the holographic principle, only time will tell.

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